Nonreciprocal Linear Transmission of Sound in a Viscous Environment with Broken *P* Symmetry

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Reciprocity is a fundamental property of the wave equation in a linear medium that originates from time-reversal symmetry, or *T* symmetry. For electromagnetic waves, reciprocity can be violated by an external magnetic field. It is much harder to realize nonreciprocity for acoustic waves. Here we report the first experimental observation of linear nonreciprocal transmission of ultrasound through a water-submerged phononic crystal consisting of asymmetric rods. Viscosity of water is the factor that breaks the *T* symmetry. Asymmetry, or broken *P* symmetry along the direction of sound propagation, is the second necessary factor for nonreciprocity. Experimental results are in agreement with numerical simulations based on the Navier-Stokes equation. Our study demonstrates that a medium with broken *PT* symmetry is acoustically nonreciprocal. The proposed passive nonreciprocal device is cheap, robust, and does not require an energy source.

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A source of sound may generate a quite complicated pattern of pressure in an inhomogeneous medium. The acoustodynamic field can be calculated analytically only for a few simple arrangements of scatterers. For more complicated geometries, one relies on numerical solutions. In a linear and lossless medium, the accuracy of the solution can be controlled via Rayleigh's reciprocity theorem, which states that a signal emitted by a source at a point *A* and received at a point *B* remains the same if the positions of the emitter and receiver are switched [1]. Two common concepts of nonreciprocity in sound propagation are based on nonlinear effects [2,3] and on local circulation of fluid [4,5]. They originate from two known exceptions when Lorentz's and Rayleigh's reciprocity theorems become invalid due to the breaking of time-reversal symmetry.

The reciprocity theorem is very general, since it originates from the time-reversal symmetry of the wave equation. It is valid for anisotropic media, for media with temporal dispersion, and even for media with dissipative losses [1,6,7]. At first glance, the latter statement contradicts the irreversibility of any process accompanied by an increase of entropy. However, the process can be irreversible but still reciprocal if the energies dissipated for forward and backward propagation are equal. Wave transmission through a medium with energy losses becomes nonreciprocal if dissipation changes with the direction of propagation. Recently this property was explored to demonstrate that acoustical losses may serve as a source of *T* symmetry violation, thus leading to nonreciprocity in the diffraction of sound from a gradient-index metasurface [8].

Dissipation in a viscoelastic medium is usually introduced by adding the imaginary part to the modulus of elasticity [9]. This leads to exponential decay of the wave intensity, but the energy losses accumulated for the opposite directions of propagation remain equal. Indeed, the decaying solutions are irreversible, but they are the solutions of the *reciprocal* wave equation. This is the physical reason why the reciprocity turns out to be compatible with dissipation. In recent reviews on the nonreciprocal propagation of sound [10–12], as well as in the mathematical proof of the reciprocity theorem [7], the statement regarding reciprocal propagation in dissipative media is related to the particular class of media with complex elastic moduli.

Complex (or dynamic) elastic modulus is a phenomenological parameter which is introduced in the macroscopic approach. A more detailed (microscopic) approach requires calculation of the field of velocities $\mathbf{v}(\mathbf{r})$ generated by a propagating sound wave. The power dissipated due to viscosity is obtained by integration of the local gradients of velocity [1]

$$\dot{Q} = -\int \left[\frac{\eta}{2}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3}\delta_{ik}\frac{\partial v_l}{\partial x_l}\right)^2 + \xi(\nabla\cdot\mathbf{v})^2\right] dV. \quad (1)$$

Here η and ξ are the viscosity coefficients. Integration runs over the volume occupied by viscous fluid. We assume that the scatterers are solid objects where dissipation can be neglected. The vector field of velocities $\mathbf{v}(\mathbf{r})$ in a viscous fluid is calculated from the Navier-Stokes equation solved together with the continuity equation [1]. For sound waves, these equations can be linearized, leading to the following equation for velocity component $v_i(\mathbf{r})$:

$$\rho\ddot{v}_i - \frac{\partial}{\partial x_i}(\lambda\nabla\cdot\mathbf{v}) = \frac{\partial}{\partial x_k}\left[\eta\left(\frac{\partial\dot{v}_i}{\partial x_k} + \frac{\partial\dot{v}_k}{\partial x_i} - \frac{2}{3}\delta_{ik}\nabla\cdot\dot{\mathbf{v}}\right)\right] + \frac{\partial}{\partial x_i}(\xi\nabla\cdot\dot{\mathbf{v}}), \qquad i = x, y, z. \qquad (2)$$

Here $\rho = \rho(\mathbf{r})$ is the mass density, and $\lambda = \lambda(\mathbf{r})$ is the bulk elastic modulus of the fluid.

Equation (2) is obviously nonreciprocal, since the terms on the right-hand side contain the derivative $\dot{\mathbf{v}} = \partial\mathbf{v}/\partial t$, which changes its sign under time reversal. The reciprocity theorem does not hold for this equation. However, the nonreciprocity is not manifested in the very special case of a symmetric set of scatterers along the direction of propagation. The decay of sound in this case is exactly the same for forward and backward directions; thus the effect of nonreciprocity turns out to be hidden by geometrical symmetry. Unlike this, in a general asymmetric case the vector field $\mathbf{v}(\mathbf{r})$ and the energy absorbed depend on the direction of propagation of sound, giving rise to nonreciprocity. Any asymmetric scatterer(s) is a source of nonreciprocity. But in order to make the effect stronger, the symmetry must be essentially broken. The dissipation increases in the regions with strong gradients of velocity. Therefore, scatterers with sharp corners are more suitable for the experimental demonstration of nonreciprocity due to gradient induced differential dissipation (GIDD).

For experimental demonstration of nonreciprocal transmission due to GIDD, a phononic crystal of aluminum rods in water environment was used. A sample, shown in Fig. 1, has a square unit cell with parity symmetry (P symmetry) broken along the vertical y axis. A unit cell in Fig. 1(c) remains invariant under parity transformation ($x \rightarrow -x$, $y \rightarrow y$), but it is not invariant under the complementary transformation ($x \rightarrow x$, $y \rightarrow -y$). Both transformations correspond to parity inversion, since they are represented by 2×2 matrices with determinant -1. Thus, by measuring acoustic transmission along two perpendicular directions (along the x and y axes) in this 2D phononic crystal, one can conclude about the role of P symmetry in reciprocal or nonreciprocal propagation of sound.

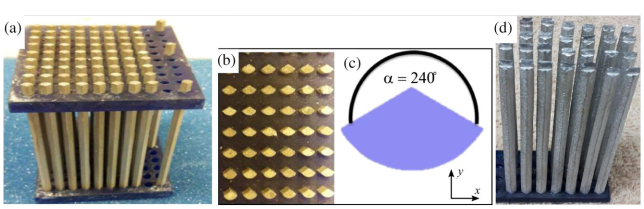

FIG. 1. Phononic crystals used for the measurements of acoustic transmission. (a) General view of the sample with anodized rods. (b) Top view. The P symmetry is broken along the vertical axis, and it holds along the horizontal axis. (c) Square unit cell with asymmetric scatterer. The angle α is a measure of broken P symmetry. (d) Sample with 4×7 rows of unanodized aluminum rods.

The period of the phononic crystal lattice is $a = 5.5$ mm, and the radius of the 120° circle sector is 2.2 mm. Two V301 1″ Panametrics 0.5 MHz immersion transducers in a bistatic setup were arranged to measure forward and backward transmission. More details about the samples and their fabrication can be found in the Supplemental Material [13–16].

First, the transmission was measured along the symmetric direction. The measured spectra for forward and backward transmission are given by two colored lines in Fig. 2. The black line in Fig. (2) shows the transmission spectrum simulated by COMSOL software. Both experimental spectra in Fig. 2 show most of the signatures obtained numerically. The calculated transmission exhibits a peak at $f = 398$ kHz which fits the gap region. This peak is due to constructive interference between a finite number of rows. Results obtained for longer samples show that with increasing length this peak is shifted towards the passing band, its amplitude quickly decreases, and the transmission within the gap vanishes. Due to the inverse symmetry along the direction of sound propagation, the transmission does not exhibit any regular feature of nonreciprocity. Small fluctuations in the spectra are typical for this type of measurement. The sound waves experience anisotropic scattering at each rod, but they follow the same "path" propagating forward and backward. Since the simulated spectra are exactly the same for two opposite directions, only one black line appears in Fig. 2. Thus, the propagation along the direction with P symmetry is reciprocal.

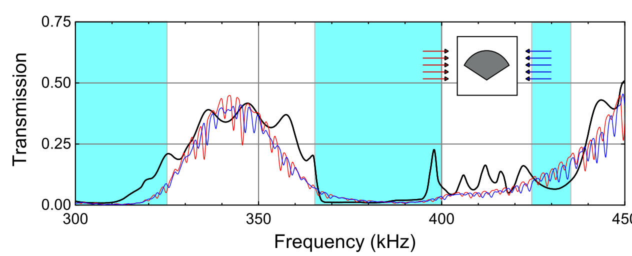

FIG. 2. Spectra of reciprocal transmission for the phononic crystal of anodized rods. Two measured spectra of transmission along the direction with P symmetry (red and blue lines) are practically equal; i.e., the transmission is reciprocal. The black line is the transmission spectrum simulated in COMSOL software. Shaded regions show the positions of the band gaps calculated for an infinite sample. The insert shows the orientation of the unit cell with respect to the direction of the incoming wave.

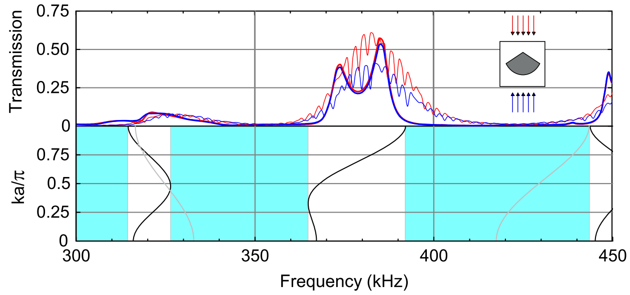

FIG. 3. Band structure and spectra of nonreciprocal transmission for the phononic crystal of anodized rods. *Lower panel:* Band structure of an infinite phononic crystal with inviscid water background for a sound wave propagating along the direction of broken P symmetry. Passing bands corresponding to even (odd) eigenmodes are shown by black (grey) lines. Regions of gaps between the even zones are shaded. *Upper panel:* Wavy lines show experimental spectra for sound waves propagating forward (thin red line) and backward (thin blue line). Numerically calculated transmission spectra are shown by smooth thick lines of the same colors. The insert shows the orientation of the unit cell with respect to the direction of the incoming wave.

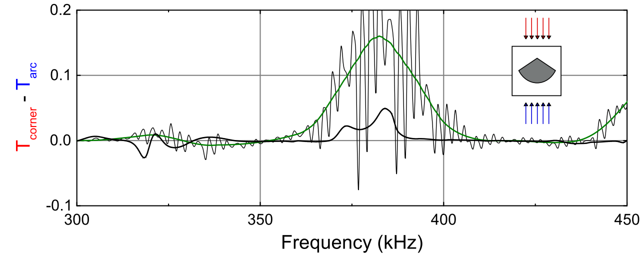

FIG. 4. The nonreciprocity in the transmission spectra of the phononic crystal of anodized rods; the difference between the transmissions coefficients plotted in Fig. 3. Experimental (numerical) data are shown by a thin (thick) line. The green line is the result of averaging over fast oscillations.

Unlike this, the transmission spectra for propagation along the line of broken P symmetry exhibit regular features of nonreciprocity. In Fig. 3, experimental (thin lines) and numerical results (thick lines) for the transmission in two opposite directions are plotted together. Theoretical and experimental results are in reasonable agreement. All the gaps and passing bands of the band structure in Fig. 4 are seen in the measured transmission spectra. At normal incidence only the even modes—i.e., the modes that are symmetric over the vertical axis—can be excited. These even modes are shown by black lines in the band structure in Fig. 3. The odd modes turn out to be deaf at normal incidence, and they are shown by grey lines. Due to asymmetry of the scatterers, there are relatively large gaps (shaded in Fig. 3) between the even passing bands. Within the band gaps, the transmission loss reaches upwards of 20 dB. Both experimental and numerical results show relatively high transmission within the gap region for frequencies $326 < f < 335$ kHz. We attribute this to excitation of the odd eigenmode existing in this frequency range. This becomes possible because the acoustic beam radiated by a finite-size vibrating membrane has, of course, some Fourier components with nonzero wave vectors in the horizontal direction. These diffracted components may excite the odd mode. The experimental transmission drops relatively slowly within the gap with the edge at $f = 392$ kHz. Finite transmission extends up to 410 kHz. It is due to dissipation, which smooths the edges of the gaps and leads to the final density of states within the gaps [17]. Since dissipation increases with frequency, the narrow gap between 425 and 435 kHz in Fig. 2 is not well resolved.

Qualitatively, the nonreciprocity is characterized by the difference $T_{\mathrm{corner}} - T_{\mathrm{arc}}$ between the acoustic energy transmitted through the phononic crystal when the incoming wave hits the corner (red line in Fig. 3) and the rounded part (blue line) of the rods. This difference is plotted vs frequency in Fig. 4. The experimental curve exhibits fast oscillations which originate from weak irregular fluctuations of the transmitted energy in Fig. 3. The green curve in Fig. 4 is obtained by averaging over these oscillations. The nonreciprocity corresponding to the numerically calculated transmission is shown by the black thick line. Nonreciprocity is reduced in the regions of gaps where the transmission is low. While there is a general agreement between the theory and experiment, it is clearly seen that the measured nonreciprocity exceeds the numerically simulated one. We attribute this difference to microscopic roughness of the aluminum rods. The rods were anodized to increase their resistance against oxidation in water. It is known that the surface of an anodized sample may have roughness of the size order from a few to dozens of microns. At this scale, the surface of the rods is not flat. Driven by oscillating sound pressure, viscous fluid slows down near the surface of a rod at a typical distance of $\delta = 2\pi\sqrt{2\eta/(\omega\rho)}$. At the frequencies $\omega \sim 10^6\ \mathrm{s}^{-1}$, the thickness of the viscous boundary layer in water (Stokes boundary layer) is estimated to be ideally about a few microns. Since the essential part of acoustic energy dissipates within the boundary layer δ, the micron-size roughness strongly affects the level of dissipation. Roughness not only changes fluid dynamics within the boundary layer; it also increases the effective area where the energy dissipates. Thus, surface roughness increases the dissipation of sound energy that leads to stronger nonreciprocity. Random roughness also can be considered as a stochastic element of the system that breaks the P symmetry at the microscopic level. At the same time, the micron-size roughness does not contribute to scattering, because the wavelength of sound in water is about 4–5 mm.

The effect of nonreciprocity can be demonstrated not only in the transmitted power, but also in the dissipated power $\dot{Q}$ given by Eq. (1). The distribution of velocities $\mathbf{v}(\mathbf{r})$ was calculated for a set of frequencies from 300 to 450 kHz. The gradients of all components of velocity were calculated over the region occupied by the sample, and the integral (1) was calculated for each frequency. The result of these calculations is presented in the Supplemental Material [13]. While the energy dissipated in the sample is small, it far exceeds the energy dissipated within an equal volume of free water. The viscous decay length of 300 kHz sound in free water is about 100 m. Sound waves propagating through a phononic crystal decay much faster. This occurs due to multiple reflections from solid surfaces of the rods. Each reflection is accompanied by high absorption [1] with the rate $\sim\sqrt{\eta\omega}$. The dissipation of acoustic energy in a phononic crystal can be calculated using perturbation theory over the terms proportional to the viscosity coefficients in Eq. (1). After quite long calculations, the linear correction $\Delta\omega_n(\omega, \mathbf{k})$ to the nth eigenfrequency $\omega_n(\mathbf{k})$ of a lossless phononic crystal can be expressed through multiple sums over reciprocal lattice vectors [13]. Evaluating the imaginary part near the frequency $f_0 = 373$ kHz, where $|\dot{Q}|$ has a local maximum, we obtain that the decay length $1/\mathrm{Im}k = |V_g(f_0)|/\mathrm{Im}\Delta\omega_2(f_0)$ does not exceed 10 m. Here $\mathbf{V}_g = \partial\omega/\partial\mathbf{k}$ is the group velocity. This decay length is an order of magnitude less than that in free water. In fact, the decay length is probably even less due to surface roughness. A decrease of the decay length due to the presence of solid scatterers was predicted in Ref. [18], where the effective viscosity has been introduced in the long-wavelength limit. Calculated in Ref. [13], correction $\Delta\omega_n(\omega, \mathbf{k})$ opens a way to introduce the effective viscosity for a 2D phononic crystal at any frequency.

The nonreciprocity in the spectra shown in Fig. 3 is quite weak, achieving a maximum of about 5 dB. It cannot be strong, because it originates from the difference between two quantities (acoustic absorption), and each one is weak by itself. Indeed, the length of the sample is ∼10 cm, and the decay length of sound is ∼10 m. Stronger nonreciprocity requires higher levels of viscous dissipation. The latter can be increased not only by increasing the viscosity of the background fluid but also by using rods with rougher surfaces. To demonstrate stronger nonreciprocity, we used a phononic crystal with the same parameters as shown in Figs. 1(a)–(c), where anodized aluminum rods are replaced by unanodized rods; see Fig. 1(d). These unanodized rods were formed using investment casting in a mold, and their surfaces are of much lower quality than those of anodized rods. The transmission spectra for propagation along the direction of broken P symmetry are shown in Fig. 5 in linear and logarithmic scales. For this shorter (4×8) sample, the details of the band structure are not well manifested because of much stronger dissipation. Here the nonreciprocity reaches 10–15 dB; i.e., it is much stronger than was observed for the anodized sample. The wave that propagates towards the sharp corner of the rods is strongly suppressed as compared to the reversed wave. Such a level of nonreciprocity allows the rectification of acoustic signals, while it still remains lower than that

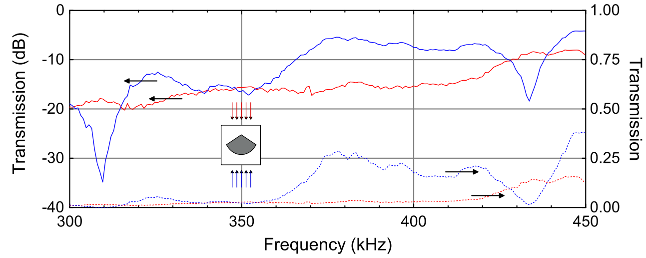

FIG. 5. The transmission spectra of the phononic crystal of unanodized rods shown in Fig. 1(d). Solid lines show the transmission plot in a logarithmic scale with the left vertical axis. Dotted lines show the linear transmission with the right vertical axis. The nonreciprocity level is about 10–15 dB within the transmission bands. The insert shows the orientation of the unit cell with respect to the direction of the incoming wave.

reported in Refs. [2,4], where acoustic nonreciprocity was achieved by either nonlinearity or air-flow bias. An important advantage of the proposed device is the broadness of the band of nonreciprocal transmission. It turns out to be orders of magnitude wider than the bands of nonreciprocal transmission of the earlier reported devices.

A periodic distribution of asymmetric scatterers enhances the effect of nonreciprocal propagation of sound. However, even a single asymmetric scatterer is sufficient to break PT symmetry and observe nonreciprocity. We calculated the nonreciprocity measured by the quantity $p_B(A)/p_A(B) - 1$. For the pressures produced by two equal quasipoint sources radiated at 10 MHz and located at A and at B, see Fig. 6. The reciprocal theorem states that for inviscid fluid, $p_A(B) = p_B(A)$ for any shape of the scatterer [1]. Viscosity and asymmetry give rise to nonreciprocity, i.e., $p_A(B) \neq p_B(A)$. For the viscosity of water and the size of the scatterer used in the phononic crystal in Fig. 1, the nonreciprocity is very weak, requiring a high accuracy of numerical calculations. The error in the numerical data in Fig. 6 does not exceed 1%. To

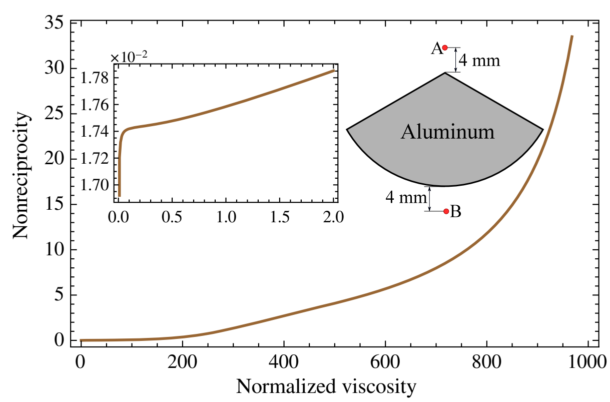

FIG. 6. Gradual growth of nonreciprocity with viscosity. Numerically calculated measure of nonreciprocity $p_B(A)/p_A(B) - 1$ vs normalized viscosity for frequency 10 MHz. Insets show the geometry of the problem and a blowup of the region of low viscosities.

demonstrate that $p_B(A)/p_A(B) - 1 \neq 0$, this difference is plotted for increasing values of the viscosity normalized to viscosity of water. The graph in Fig. 6 shows a graduate increase of $p_B(A)/p_A(B) - 1$ that serves as direct evidence of nonreciprocity induced by broken PT symmetry.

In conclusion, a new mechanism of nonreciprocal acoustic transmission through a medium with broken PT symmetry is presented. Since the violation of time-reversal symmetry is due to finite viscosity, the propagation of sound is described by the Navier-Stokes equation. Unlike the widely used approach where dissipation is introduced through complex elastic moduli, viscous fluid dynamics leads to truly nonreciprocal propagation of sound if inversion symmetry is broken. The proposed mechanism can be observed using a passive linear device—phononic crystals with asymmetric scatterers. Using this passive device, which does not require an external source of energy, nonreciprocity is observed within very wide ranges of frequencies. It is demonstrated that the level of nonreciprocity increases for scatterers with rough surfaces, which means that the effective viscosity can be tuned by changing the quality of the surface of the scatterers. The observed nonreciprocal transmission is a finite-size effect. It vanishes for very long samples, since the transmission becomes exponentially small.

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